Magnetic and Winding Loss Estimation Using Harmonic Binning

Reanna Orzechowski, Matthew Jahnes, Youssef Fahmy, Liwei Zhou, Matthias Preindl
Dept. of Electrical Engineering, *Columbia University*, New York, NY USA
Email: reanna.o@columbia.edu, matthias.preindl@columbia.edu

***Abstract*—The current through the magnetics in a non-resonant topology is typically a composition of multiple frequencies and can be considered harmonic-rich. This is increasingly true in phase-shifted topologies where the current through the magnetic device is not strictly a sinusoidal wave. This paper follows the calculation of the transformer losses in a phase-shifted topology using the proposed harmonic binning approach. We will show that using the harmonic binning components of a triangular wave magnetic field signals allows us to use the Steinmetz equation in a fast and simulation-friendly way. First, a brief discussion on the theory behind winding and core loss is presented followed by a discussion of analytical transformer modeling. Then the harmonic binning method for winding and core loss is introduced. Lastly, FEA model results and experimental results for a transformer are presented. The proposed method is close to the FEA and experimental loss model results.**

***Index Terms*—magnetics, loss modelling, steinmetz**

I. Introduction

There are two main drivers of loss in modern power converters: losses in a magnetic device (inductor and transformer) and losses in a switching device. Losses in switching devices are a highly studied field [1]–[3]. This paper pertains to the calculation of losses within the magnetic devices including the losses within the core and the losses within the winding of the magnetic device.

Ferro-magnetic and ferri-magnetic materials which are generally used as transformer core materials, will dissipate heat when exposed to time-varying magnetic fields. Core losses are composed of two mechanisms: hysteresis and eddy currents. Hysteresis occurs from the fact that flux does not move effortlessly through a magnetic material, but rather requires work. For an AC magnetic field applied to a magnetic material, assuming the hysteresis curve is independent of the frequency of the applied field, average power loss per volume for one cycle is given by

$$\frac{P_h}{V} = \oint H\, dB f \tag{1}$$

The eddy current loss mechanism stems from currents induced by the time-varying magnetic flux densities. By Faraday's Law, varying flux densities induce electric fields in a material, which will generate a current if the material is electrically conductive. These currents then induce winding losses in the material.Charles Steinmetz created an empirical model that captures the hysteresis and eddy current effects of core losses. The famous Steinmetz equation gives the core loss per unit volume for the sinusoidal input of of a core is famously

$$P_v = kf^{\alpha}B^{\beta}, \tag{2}$$

where k, α, and β are curve fitted constants that depend on the core material and operating frequency. To calculate the total core loss from equation 2 one can multiply by total volume of the core.

Winding losses are due to the heat dissipated in the wires that are wound around the transformer from a current. As work is performed by movement of electrical charge through a potential difference, the electrical power dissipated is determined by differentiating electrical work with respect to time.

$$P_{elec} = \frac{dw_{elec}}{dt} = v\frac{dq}{dt} = vi \tag{3}$$

It is important to know the extent of losses within magnetic devices as loss modeling and optimization are necessary steps in high-efficiency and/or high power density converter design [4], [5]. It is often the case that, during converter design, loss in one area is traded for loss in another i.e. using a large inductance to lower switching loss will increase the inductor loss. For an optimal converter design, losses must be balanced, and this is only achievable through loss modeling.

Modeling the losses of magnetic cores is a notoriously challenging task with some preferring the use of finite-element analysis (FEA) [6]–[8], citing that while others use the Steinmetz equation and its derivatives [9]–[13]. This paper falls into the latter, where the Steinmetz equation is used in conjunction with Fourier analysis to develop a more accurate first-order approximation core loss modeling approach.

II. Analytical Transformer Modeling

This section will discuss the modeling of a transformer in the magnetic and electrical domain. While the design of the core type and wire type is out of the scope of this paper, we will describe the analytical equations that describe key parameters of a transformer for a given design. The ability for the primary side winding of a transformer to induce a magnetic flux in the core that produces a voltage on the secondary side of the transformer is known as the magnetizing inductance. For an ideal transformer, the magnetizing inductance is infinite and all of the flux from the primary side of the core is transferred to the secondary side. In realistic transformer applications, there is an amount of flux that does not get transferred from one side

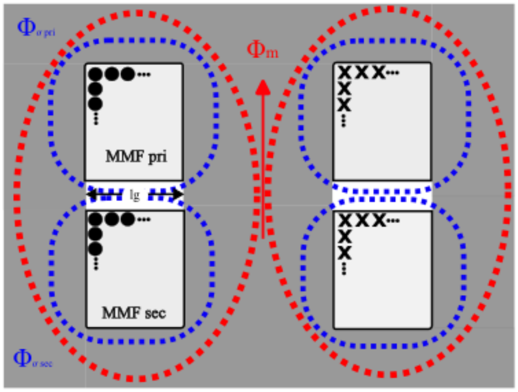

Fig. 1. Magnetic model of transformer

of the transformer to the other. The flux that "leaks" between the primary and secondary sides is referred to as the leakage flux. In phase-shifted converter designs, the leakage inductance of the transformer is a design parameter as it allows power to flow from one side of the circuit to the other. In phase-shifted topologies that aim to achieve a certain output power with a given switching frequency, the transformer design is strategically planned to attempt to achieve a certain leakage inductance. As resistance is to the electrical domain, reluctance is to the magnetic domain. The reluctance of a given core can be calculated based on its permeability, μ_r, magnetic path length, l_m and cross-sectional area, A_e as

$$\mathscr{R}_{core} = \frac{l_m}{\mu_0 \mu_r A_e}. \tag{4}$$

Similarly, the reluctance of the leakage flux is based solely on permeability of air and the magnetic path length of the leakage path. The reluctance of the gap, is

$$\mathscr{R}_{gap} = \frac{l_g}{\mu_0 A_e}. \tag{5}$$

With the reluctance calculated based on the core material parameters, the corresponding inductances relate directly to the number of turns of the transformer and the reluctance.

The magnetizing inductance for number of primary turns, N_p and reluctance, R_m is

$$L_m = \frac{N_p^2}{\mathscr{R}_{core}}. \tag{6}$$

The leakage inductance is similarly

$$L_{\sigma\ primary} = \frac{N_p^2}{\mathscr{R}_{gap}} \tag{7}$$

The magnetic flux in a core is related to the magnetizing inductance L_m, the number of primary turns, N_p, the cross-sectional area, A_e, and the current, i_p by the relation

$$\Phi_M = \frac{L_M i_p}{N_p}. \tag{8}$$

The corresponding magnetic field can be found from Φ_M by

$$B_M = \frac{\Phi_M}{A_e}, \tag{9}$$

meaning that the magnetic field through the core is related to the current. Fig. 1 shows a model of a transformer with a primary and secondary winding that produces a magneto-motive force (MMF) when a current is passed through and the corresponding magnetizing and leakage flux that is induced in the core as a result.

Now that the inductance parameters of the transformer are defined, an equivalent circuit can be constructed. As this is a circuit in the electrical domain, we can measure the currents through the leakage and magnetizing inductance and then calculate what the corresponding flux and magnetic field. An equivalent circuit model of a transformer with a finite magnetizing inductance and primary and secondary leakage is shown in Fig. 2.

III. Loss Estimation Methods

This section will discuss how to model the two main losses within magnetic devices: winding (also referred to as copper) losses and core losses. The magnetic loss method presented in this paper was created with the goal of being simple, computationally fast, and quickly implementable.

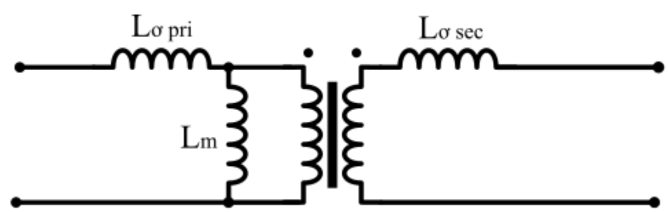

Fig. 2. Equivalent circuit model of transformer

A. Winding Loss

For a wire with resistance, R and current, i the electrical power loss is $P_{elec} = i^2 R$. For time-varying excitation, the resistance of a wire varies with frequency. The re-distribution of current over a conductor's cross-section causes the ratio of $\frac{R_{ac}}{R_{dc}}$ to change because certain portions of the conductor are not fully effective in carrying current. For an isolated conductor, the ratio $\frac{R_{ac}}{R_{dc}}$ is given by a function of the parameter x defined as

$$x = \sqrt{\frac{8\pi\mu f A}{\rho \times 10^9}} \tag{10}$$

Where A is the wire area in sq cm, d is the wire diameter in cm, μ is the permeability of the conductor. ρ is the specific resistivity in ohms per cubic centimeter, f is the frequency and R is the resistance to direct current for 1 cm of conductor in ohms. Then using Table 4 in [14] the ratio of alternating-current resistance to direct-current resistance for a solid round wire can be found as a function of x.

The Litzendraht, or *litz* conductor, contains a large number of individual fine wire strands that are insulated from each other except for at the ends where the wires are connected in parallel. The ratio of AC to DC resistance of an isolated *litz* wire is given by the Ternan equation [14], [15]

$$\frac{R_{ac}}{R_{dc}} = H + k\left(\frac{nd_s}{d_o}\right)^2 G, \tag{11}$$

where H is the resistance ratio of the individual strand when isolated, G is the constant taking into account proximity effect of neighboring wires, n is the number of strands in the cable, d_s is the diameter of individual strand. D_o is the diameter of cable and k is a constant depending on n which is equal to 2 for any n greater than 27. For a given operating frequency and current excitation, this resistance factor can be used to calculate the corresponding winding losses.

B. Core Loss

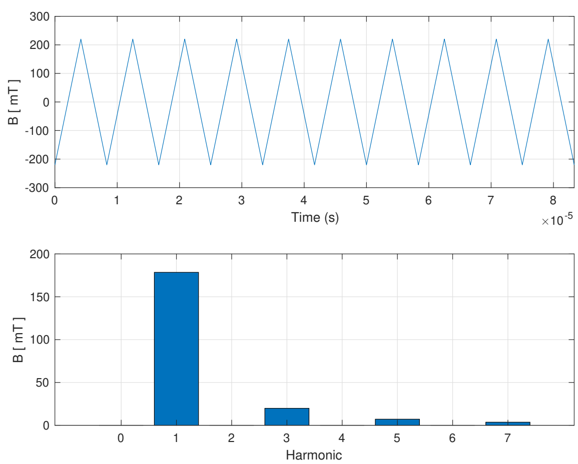

Fig. 3. *PLECS* simulation result of magnetic field and corresponding Fourier series showing the frequency harmonic components

The harmonic binning loss estimation method proposed in this paper is simulated in PLECS. First a transformer model is built in PLECS with a certain turns ratio, leakage, and magnetization inductance. The magnetic flux through the transformer core is measured, then the Fourier series is computed giving the multiple frequency harmonic components. For a triangular wave with period 2L, the Fourier series, f(x), is given as

$$f(x) = \frac{8}{\pi^2}\sum_{n=1,3,5,\ldots}^{\infty} \frac{(-1)^{(n-1)/2}}{n^2}\sin\left(\frac{n\pi x}{L}\right) \tag{12}$$

Our method is to use the Fourier series to decompose a non-sinusoidal signal into its sinusoidal parts and then apply the Steinmetz equation at each harmonic with the corresponding frequency and peak magnetic field density. Finally, the total core loss is the sum of all the individual contributions at each decomposed harmonic. The empirical core loss density $[mW/cm^3]$ as a function of operating temperature (T in ºC), frequency (f in Hz), flux density, (B in T) is given from the Ferroxcube manufacturer as

$$P_v = Cmf^xB^y(Ct_2T^2 - Ct_1T + Ct) \tag{13}$$

The constants Cm, x, y, Ct_1,Ct_2, and Ct are given by Ferroxcube as an estimation of typical material performance.

The Steinmetz equation calculates the core loss by using the appropriate Steinmetz coefficients at each frequency harmonic. The fundamental harmonic is the switching frequency of the converter, and the higher order harmonics are integer multiples of this switching frequency. For example, for a switching frequency, F_{sw}, the 2nd harmonic is at the frequency $2 * F_{sw}$. Further, the Steinmetz coefficients change as a function of frequency, so the higher-order harmonics usually have different co-efficient values. The higher-order harmonics still have some magnetic-field component but it is lesser in magnitude and the Steinmetz coefficients are smaller and thus have less effect. The loss contributed at each harmonic frequency is summed together to express the total power loss. Fig. 3 shows the time varying magnetic field through the core on the left and the de-composed Fourier series of the triangular waveform on the right ordered by harmonic. Since there are 7 harmonics in this example, we apply the Steinmetz core loss equation 7 times noting that the 2nd, 4th, and 6th harmonics contribute no magnetic flux in the core. The signal in the time-domain is even, so there is not a 0th (DC-offset) harmonic component seen in the frequency domain. This paper assumes that using the first seven harmonics of the Fourier series are relevant to get accurate core loss results, although this method could be extended to as many harmonics is desired.

What works about this method is that the Fourier series of a triangle wave is multiple sinusoidal waves. The Steinmetz equation was created to model core loss due to sinusoidal inputs and by using the harmonic binning we are applying exactly that. Although this method assumes that the frequency components are linear and independent from one-another, an assumption which is generally not true, we figure this method will give reasonably accurate results and has the advantage of being integrated into a simulation environment where other loss components such as switching losses can be computed as well.

TABLE I
CORE PARAMETERS

Core Material	**Ferroxcube E42/21/20**		
	Effective Length	***Effective Area***	***Effective volume***
3F36	97 (mm)	233 (mm^2)	22700 (mm^3)

IV. RESULTS

To assess the accuracy of the harmonic binning model, it was necessary to compare our results to trusted methods such as FEA models and experimental measurements. The selection

TABLE II
LITZ WIRE SPECIFICATIONS

Wire Type	New England Wire (NEW)		
	Strand Number	***AWG***	***Outer Diamater***
NEW 1	2625	44	0.149 (in)

of the core type and wire type will depend on many factors such as cost, operating frequencies, and application. For high-frequency applications, the 3F36 core material was selected. We chose to assemble two EE-cores in parallel to double the effective area and increases the corresponding window to cross-sectional area product, allowing for higher power transfer. Litz wire was chosen for its robustness to winding loss, as discussed earlier in this paper. The litz and core parameters relevant to this paper are detailed in Table I and II.

FEA models of transformers in programs such as ANSYS Maxwell discretely models and solves meshes within the 3-D component using advanced mathematical solvers and sums the mesh results [16]–[20].The FEA model solves key parameters of the transformer such as the coupling coefficient, leakage inductance, magnetizing inductance, core and winding loss. While this method has been proven to provide accurate loss results, it does not allow the magnetic loss calculations to be integrated into a circuit simulation and design tool such as *PLECS*. An FEA model of 2 EE 42/21/20 cores with 3F36 material and NEW 1 Litz wire was created in ANSYS. The core-loss parameters for the 3F36 material were also integrated into the ANSYS model. A transient simulation with a triangular wave input to the primary side winding was run while the secondary side of the transformer was shorted. Fig. 4 shows the measured magnetic field throughout the core. To calculate core loss, we calculated the average core loss given in ANSYS over one complete switching period.

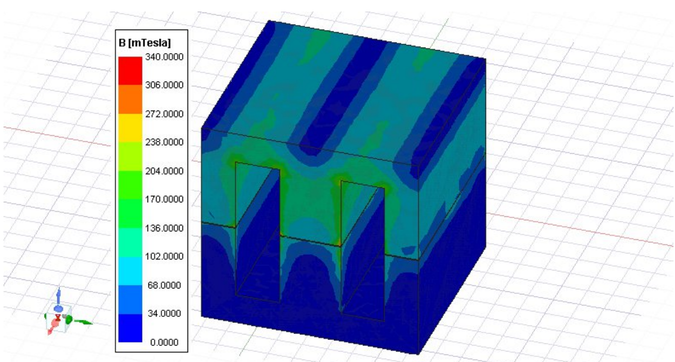

Fig. 4. Magnetic field in core of ANSYS Model

Finally, a transformer was built and its loss was measured experimentally. A complexity associated with building magnetic devices is their consistency. If there is even a small variation in the device during assembly, the magnetizing and leakage inductance of the transformer will be affected. The analytical leakage inductance is based on the number of turns and reluctance of the air gap. The magnetizing inductance is based on the number of turns, reluctance of the core, and reluctance of the air gap. In general, it is very hard to build a transformer without any air gap, as oftentimes air gap is a by-product of transformer assembly. This is why the analytical and ANSYS calculations for magnetizing inductance have a range of values, as shown in Fig. 5. The magnetizing inductance was calculated assuming best and worse case air gap scenarios.

Three 1:1 transformers were built and their leakage and magnetizing inductance was measured on an LCR meter with input frequencies ranging from 10 kHz to 1 MHz. Overall, the leakage inductances of the three transformers are very close to the analytical value and very close to the ANSYS model. This shows that the ANSYS model is an accurate representation of the transformer design. The magnetizing inductance of the three transformers is generally in the range of what is expected, although the magnetizing inductance of Transformer 1 and 3 shown in Fig. 5 is slightly below the analytical value. This could be due to a slightly larger air gap in the core than was planned for. The reason the magnetizing inductance increases as the frequency approaches 1 MHz is due to the resonance of the inductance and parasitic capacitance in the winding of the transformer. As the LCR meter input frequency approaches the resonant frequency of the transformer, we see the magnetization inductance increase.

Next, the transformer was placed in a dual-active half-bridge circuit with a phase-shifted controller that applied a duty cycle of 0.5 and a switching frequency of 120 kHz. With an 800 V bus, the transformer was experimentally tested at 10 kW. Fig. 6 shows the measurements of leakage inductance current, magnetizing inductance current, instantaneous power as well as the phase-shifted voltage input to the primary and secondary sides of the transformer. With a properly calibrated oscilloscope, we measured the instantaneous power across the primary and secondary sides of the transformer. The reason the oscilloscope needs to be calibrated is so the voltage and current measurements of the transformer are aligned. If they are not aligned the instantaneous power measurement will not be accurate, leading to inaccurate loss results. Finding the instantaneous power difference between the two sides and taking the average over one switching cycle we computed the average power lost in the transformer. We broke down the components of magnetic loss for each method and summarize our findings in Fig. 7. The comparison of total magnetic losses us shown in Fig. 8. Overall, the FEA model and harmonic binning method are very similar in terms of overall loss calculations. Experimentally, we are seeing more loss than computed analytically. This is most likely due to the core under test heating up and approaching saturation, increasing its losses.

V. CONCLUSION

Accurate losses in magnetic devices are of high importance to achieve an optimal design. As discussed in this paper, the two components of losses that occur in magnetic devices

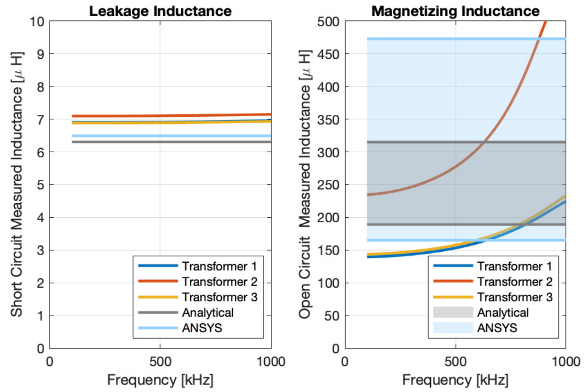

Fig. 5. Inductance measurements

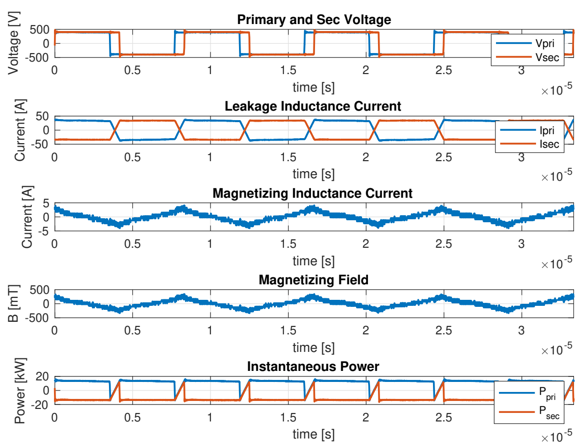

Fig. 6. Experimental measurements

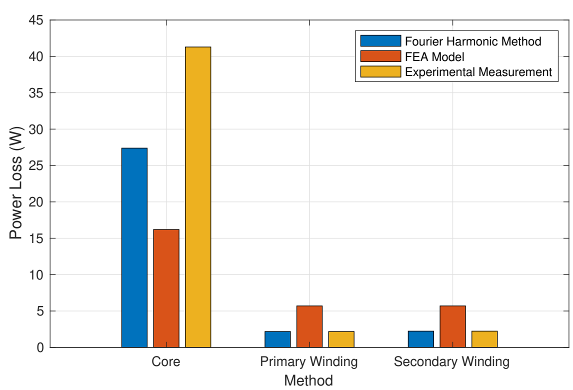

Fig. 7. Comparison of core and winding loss for FEA model, harmonic binning method and experimental results

are winding losses and core losses. We have proposed a method to analytically calculate these magnetic losses for a transformer in a circuit simulation environment by measuring the applied magnetic field and using the binned harmonic components to apply the Steinmetz equations. Generally, this

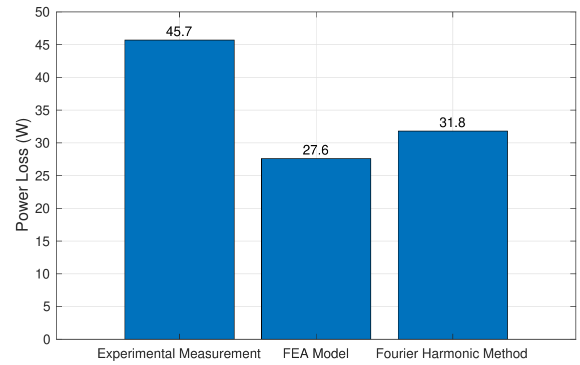

Fig. 8. Comparison of magnetic loss for FEA model, harmonic binning method, and experimental results

method is advantageous for a rapid prototype application. The results from the FEA model, experimental and harmonic binning method are relatively close. While the results in this paper focus on a triangular wave signal, the harmonic binning method is applicable to multiple types of input signals. This method is also easily applicable to a circuit simulation environment allowing for magnetic and switch loss calculations to be calculated in the same environment, making it easy to understand the overall losses for a circuit topology.

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