MTPA Fitting and Torque Estimation Technique Based on a New Flux-Linkage Model for Interior Permanent Magnet Synchronous Machines

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***Abstract*—Due to the nonlinearity of the flux-linkage profiles of the interior permanent magnet synchronous machine (IPMSM), the conventional motor model cannot be used for both maximum torque per ampere (MTPA) control and torque estimation. This paper proposes a nonlinear flux-linkage model for IPMSM with eight coefficients to fit the real d-axis flux-linkage, q-axis flux-linkage, MTPA, and torque. The corresponding torque equation and MTPA condition are presented. The factors in the proposed model can be obtained by solving an optimization problem with the limited information from the machine instead of the measurement throughout the map. The comparison of the characteristics between the proposed algorithm and FEA data is illustrated.**

I. Introduction

Interior permanent magnet synchronous machines (IPMSMs) have many advantages, such as small volume, light weight, low loss, high efficiency, high torque and power density, fast dynamic performance, etc. They have been widely used for high performance drive systems, which require quick response and wide speed range [1].

The performance of IPMSM strongly depends on the control schemes of the current vector, because selecting different current vectors leads to different torque/power. According to the characteristic of IPMSM on the iso-torque locus, a current vector can be found with the minimum phase current, which is the maximum torque per ampere (MTPA) control.

In [2]–[4], this algorithm is well explained by using the conventional motor model, which considers all the parameters of the machine, such as the d-axis and q-axis inductances, as constants. In this case, the optimal current reference can be obtained by making the derivative of the torque with respect to the current zero. However, the method based on the constant parameters in the conventional IPM motor model is not appropriate for the real machines because the d-axis and q-axis flux-linkage profiles are nonlinear. In practical applications, the development of the MTPA locus should consider the nonlinear flux-linkage profile, which is obtained from the finite element analysis (FEA) or experimental results.

In [5], MATLAB optimization toolbox is used to find the optimal current reference based on the FEA simulation results of an IPM motor. Interpolation helps this strategy obtain the best control commands beyond the avalible data from the FEA tool. But, if the machine is produced by a manufacturer, its geometry and other information might be unavailable for FEA simulations.

In [6], the conventional IPM motor model is used to decipt the nonlinearity of the flux-linkage without the machine's structure known. With the help of the measured results, a method to find the permanent-magnet flux-linkage and the d-axis and q-axis inductances at different currents under certain speed is proposed, which can offer the desired reference and fit the flux profile. Although experiments can be done to quickly search for the optimal current reference to minimize the copper loss of the motor, the estimation of the torque is still time-consuming due to the flux-likage measurement throughout the operating map. In addition, storage and manipulation of these off-line data cause another concern for engineers. A look-up-table needs to be established to store the nonlinear flux-linkage. The size of the LUT is limited due to trypical microcontroller memory constraints. This limits the acuracy of the stored torque maps.

In [7], the local linearization technique is proposed to fit the MTPA and torque by using the conventional motor model with data at certain operating points. Moore-Penrose pseudoinverse of matrix is used to solve the problem with minimum least-square errors. However, this algorithm can only be applied to the machines with small saturation. Because the nonlinear flux-likage leads to the nonlinearity of the inductances, a function with respect to the current can be established to capture the characteritic of the self and mutual inductances.

In [8], a model for the estimation of the inductances is proposed. However, it is too complicated for the derivation of the MTPA condition and torque estimation. Furthermore, this algorithm is based on the fitted inductance instead of the flux-linkage; therefore, it is indirect for the torque identification. The self-tuning MTPA strategy based on the online estimation of the motor's parameters is proposed in [9]; but, the cross-coupling effect is not considered.

In this paper, a new flux-linkage model is proposed to fit the nonlinear flux-linkage profiles so as to charaterize MTPA and estimate the torque by using the limited information of the machine. The simplicity of the proposed flux-linkage

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model results in the feasibility of the real-time implementation of the corresponding MTPA control and the torque estimation. The matching comparison between MTPA and electromagnetic torque based on the FEA data and the proposed model has been done in simulations.

II. IPMSM Model and MTPA Control

With the help of the application of the Clark and Park transformation, IPMSM can be described in dq coordinates in order to be controlled as a DC machine. The conventional IPM motor model is shown in equations (1)-(3) [2].

$$\begin{bmatrix} \Lambda_d \\ \Lambda_q \end{bmatrix} = \begin{bmatrix} L_d & 0 \\ 0 & L_q \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \begin{bmatrix} \Lambda_{pm} \\ 0 \end{bmatrix} \tag{1}$$

$$\begin{bmatrix} v_d \\ v_q \end{bmatrix} = R_s \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} \Lambda_d & 0 \\ 0 & \Lambda_q \end{bmatrix} + \omega_e \begin{bmatrix} 0 & -\Lambda_q \\ \Lambda_d & 0 \end{bmatrix} \tag{2}$$

$$\begin{aligned} T_e &= \frac{3P}{2}\left[\Lambda_{pm} i_q + (L_d - L_q) i_d i_q\right] \\ &= \frac{3P}{2}\left[\Lambda_{pm} I_a \cos\beta + \frac{1}{2}(L_q - L_d) I_a^2 \sin 2\beta\right] \end{aligned} \tag{3}$$

where, Λ_d, Λ_q, L_d, L_q, i_d, i_q, v_d, v_q are the d-axis and q-axis flux-linkages, self inductances, currents, and voltages, respetively; Λ_{pm} is the permanent-magnet flux-linkage; R_s is the stator resistance; ω_e is the electrical speed; T_e is the electromagnetic torque and β is the excitation angle between the q-axis and the phase current vector.

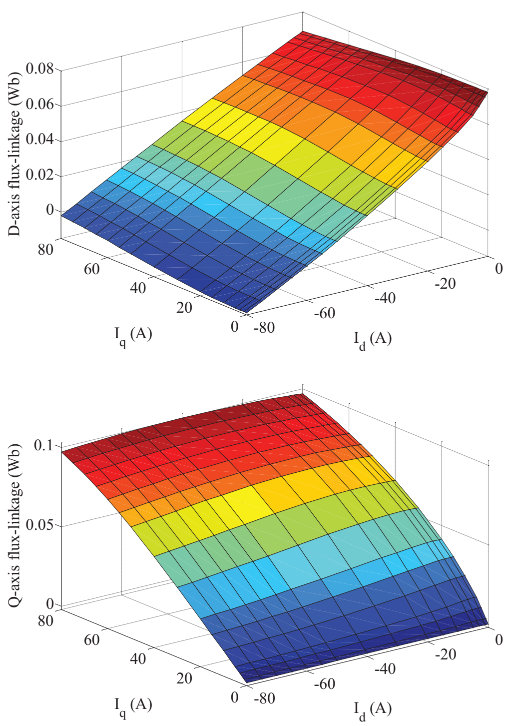

Fig. 1. D- and q-axis flux-linkage profiles based on the FEA results.

The copper-loss-minimization control can be achieved by making the derivative of the electromagnetic torque (3) with respect to the excitation angle be zero [3].

$$i_d = \frac{\Lambda_{pm}}{2(L_q - L_d)} - \sqrt{\frac{\Lambda_{pm}^2}{4(L_q - L_d)^2} + i_q^2} \tag{4}$$

Nevertheless, the flux-linkage of IPM machine is the nonlinear function of the d-axis and q-axis currents as shown in Fig. 1. Thus, the MTPA control for the practical use should be built as an optimization problem as expressed in (5) [5].

$$\begin{aligned} &\text{minimize } i_d^2 + i_q^2 \\ &\text{subject to } T_e - \frac{3}{2}P\left(\Lambda_d i_q - \Lambda_q i_d\right) = 0 \\ &\qquad i_d^2 + i_q^2 \le I_{am}^2 \\ &\qquad \left(R_s i_d - \omega_e \Lambda_q\right)^2 + \left(R_s i_q + \omega_e \Lambda_d\right)^2 \le V_{am}^2 \end{aligned} \tag{5}$$

where I_{am} and V_{am} are the maximum phase current and maximum phase voltage. MATLAB optimization toolbox is used to solve the problem. The resultant MTPA locus is shown as a solid black curve in Fig. 2.

III. MTPA Fitting and Torque Estimation Based on Conventional IPMSM Model With Constant Parameters

In order to obtain the optimal current reference in (5), the flux-linkage throughout the operating map is needed. However, this method requires the details of the machine, which may be unavailable or a large amount of the repetitive experiments is not feasible. In fact, if a set of constant parameters can be found for an IPM machine to make the known motor model have the key traits as the real machine does, the model with the appropriate approximation still can be used to provide good MTPA control and mediocre torque estimation in a simpler way.

The optimization problem shown in (6) is established for such purpose by using the conventinal IPM motor model. A set of constant d-axis inductance, q-axis inductance, and the permanent-magnet flux-linkage can be found to fit the MTPA in (4) and torque in (3). Only the data at two operating points are involved to solve (6). One is at the intersection of the maximum current circle and MTPA with nonlinear flux-linkage profiles considered in (5). The other one is located where the current circle, whose radius is half of the maximum current, and MTPA in (5) meet.

minimize:

$$\left(\Lambda_{dr} - \Lambda_{drc}\right)^2 + \left(\Lambda_{qr} - \Lambda_{qrc}\right)^2 + \left(\Lambda_{dh} - \Lambda_{dhc}\right)^2 + \left(\Lambda_{qh} - \Lambda_{qhc}\right)^2$$

$$\begin{aligned} &\text{subject to } T_{er} - \frac{3}{2} P i_{qr}\left[\Lambda_{pm} + \left(L_d - L_q\right) i_{dr}\right] = 0, \\ &\qquad \Lambda_{pm} i_{dr} + \left(L_q - L_d\right)\left(i_{qr}^2 - i_{dr}^2\right) = 0. \end{aligned} \tag{6}$$

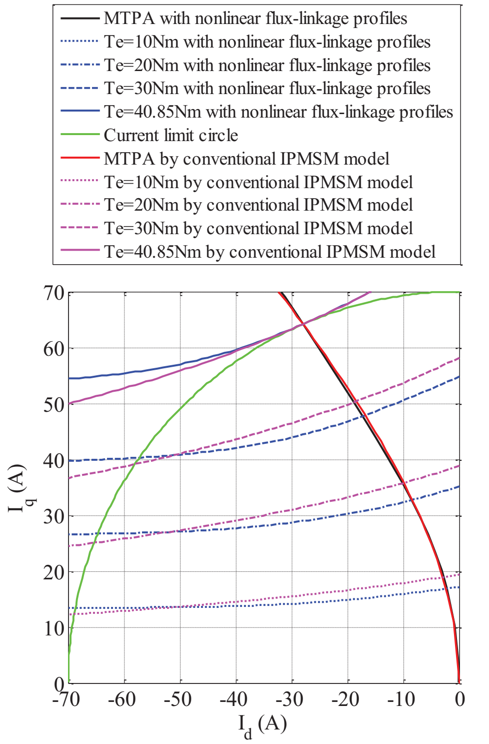

Fig. 2. Characteristic comparison between the conventional motor model and FEA data.

where T_{er}, i_{dr}, i_{qr}, Λ_{dr} and Λ_{qr} are the real torque, d-axis current, q-axis current, d-axis flux-linkage and q-axis flux-linkage at the rated operating point; i_{dh}, i_{qh}, Λ_{dh} and Λ_{qh} are the real torque, d-axis current, q-axis current, d-axis flux-linkage and q-axis flux-linkage at the intersection of MTPA control in (5) and the current circle whose amplitude is half of the current constraint; Λ_{dxc} and Λ_{qxc} are the calculated d-axis and q-axis flux-linkages based on equation (1) at different point 'x'.

The MTPA and torque estimation generated by the conventional IPM motor model with constant fitted parameters are plotted in Fig. 2. Even though the MTPA loci fit each other well, the mismatch between the identified torque in (3) with the invarient permanent-magnet flux-linkage and inductances and the one based on the FEA data is too much because of the machine's nonlinearity as shown in Fig. 3. With constant parameters, the conventional motor model is unable to accommodate itself to the real motor unless the parameters in (1)-(3) are considered as the functions of the current, which will bring too much complexity into the problem and it is not a direct way to correct the mismatch of the torque identification.

IV. MTPA Fitting and Torque Estimation Based on a New Flux-Linkage Model With Constant Parameters

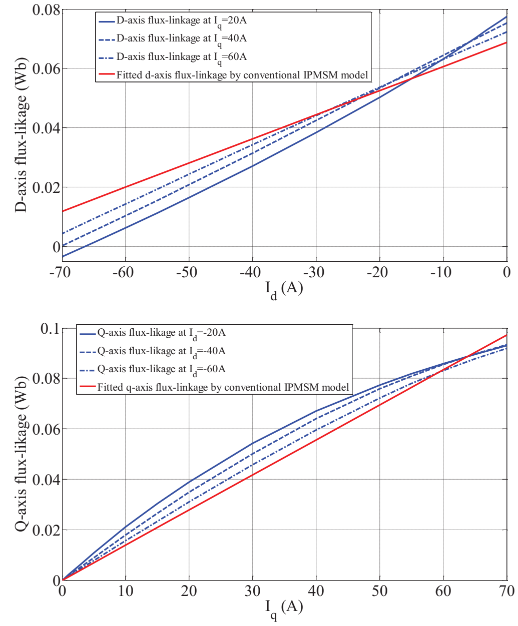

Fig. 3. D-axis and q-axis flux-linkage fittings with the conventional motor model.

Considering the saturation and cross-coupling effects on the flux-linkage profiles of an IPM motor, a new flux-linkage model is proposed as shown in (7).

$$\begin{pmatrix} \Lambda_d \\ \Lambda_q \end{pmatrix} = \mathbf{K} + \mathbf{L}\begin{pmatrix} i_d \\ i_q \end{pmatrix} + \mathbf{G} i_d i_q + \mathbf{H} i_q^2 \tag{7}$$

where $\mathbf{K} = \begin{pmatrix} \Lambda_{pm} \\ 0 \end{pmatrix}$, $\mathbf{L} = \begin{pmatrix} L_d & M_{dq} \\ M_{qd} & L_q \end{pmatrix}$, $\mathbf{G} = \begin{pmatrix} c_1 \\ c_3 \end{pmatrix}$, $\mathbf{H} = \begin{pmatrix} 0 \\ c_2 \end{pmatrix}$; M_{dq} and M_{qd} are virtual d-axis and q-axis mutual inductances; c_1, c_2 and c_3 are constant coefficients. All the parameters in matrices **K**, **L**, **G** and **H** are constants.

Compared to the conventional flux-linkage model in (1), model (7) has matrices **L** and **G** designed for the description of the influnces which the d-axis current has on the gneration of the q-axis flux-linkage and the q-axis current has on the production of the d-axis flux-linkage. As shown in Fig. 1, the q-axis flux-linkage saturates more easily than the d-axis flux-linkage does when bigger current is applied to the motor. Matrix **H** is designed for this purpose.

The corresponding torque expression can be obtained by substituting the flux-linkages in (7) into the torque equation in the flux-linkage form as shown in (5).

$$T_e = \frac{3}{2} P \begin{bmatrix} \left(L_d - L_q\right) i_d i_q + \left(c_1 - c_2\right) i_d i_q^2 \\ + \left(\Lambda_{pm} - c_3 i_d^2\right) i_q + M_{dq} i_q^2 - M_{qd} i_d^2 \end{bmatrix} \tag{8}$$

Making the derivative of the torque equation (8) with respect to the excitation angle to be zero results in the MTPA condition as shown in (9).

$$
\begin{aligned}
&-c_3 i_d^3+\left[L_d-L_q+2\left(c_1-c_2\right)i_q\right]i_d^2+\\
&\left[\Lambda_{pm}+2\left(M_{dq}+M_{qd}\right)i_q+2c_3 i_q^2\right]i_d+\\
&\left(c_2-c_1\right)i_q^3+\left(L_q-L_d\right)i_q^2=0
\end{aligned}
\tag{9}
$$

The solutions to this cubic equation [10] for the optimal d-axis current reference are shown in (10).

$$
\begin{aligned}
i_{d1}&=-\frac{1}{3a}\left(b+S+\frac{Z}{S}\right),\\
i_{d2}&=-\frac{1}{3a}\left[b+\frac{-1+i\sqrt{3}}{2}S+\frac{Z}{\frac{-1+i\sqrt{3}}{2}S}\right],\\
i_{d3}&=-\frac{1}{3a}\left[b+\frac{-1-i\sqrt{3}}{2}S+\frac{Z}{\frac{-1-i\sqrt{3}}{2}S}\right].
\end{aligned}
\tag{10}
$$

where,

$$
\begin{cases}
S=\sqrt[3]{\dfrac{O+\sqrt{O^2-4Z^3}}{2}}\\
O=b^2-3ac\\
Z=2b^3-9abc+27a^2d\\
a=-c_3\\
b=L_d-L_q+2\left(c_1-c_2\right)i_q\\
c=\Lambda_{pm}+2\left(M_{dq}+M_{qd}\right)i_q+2c_3 i_q^2\\
d=\left(c_2-c_1\right)i_q^3+\left(L_q-L_d\right)i_q^2
\end{cases}.
$$

Different sets of the coefficients lead to different roots. A simple way to identify which one is the desired reference is to substitute i_q=0A into i_{d1}, i_{d2} and i_{d3} listed in (10). In this case, the d-axis current reference is supposed to be zero in accordance with both the MTPA condition and the proposed flux-linkage model. For example, using the fitted constants in model (7) i_{d2}=0A when q-axis current is zero. Then, the optimal d-axis current can always be derived by substituting different q-axis currents into the root i_{d2} according to a set of the fitted parameters.

The top of Fig. 4 shows an IPM motor's operating range. Its maximum speed is 9000 rpm. The red curve is the MTPA locus calculated with the FEA data. On the left of the red curve, MTPA control can be applied. On the right, the flux-weakening control is dominant, where the current reference is found at the intersection of the voltage ellipse and the iso-torque locus.

In order to obtain the coefficients listed in (7), the necessary information should be involved. As shown in Fig. 4, the flux-linkage data at points 'R', 'A', 'B', 'C', 'D', 'E', 'F', torque information and MTPA fitting condition at point 'R' are selected. Points 'R', 'C' and 'E' are on the current

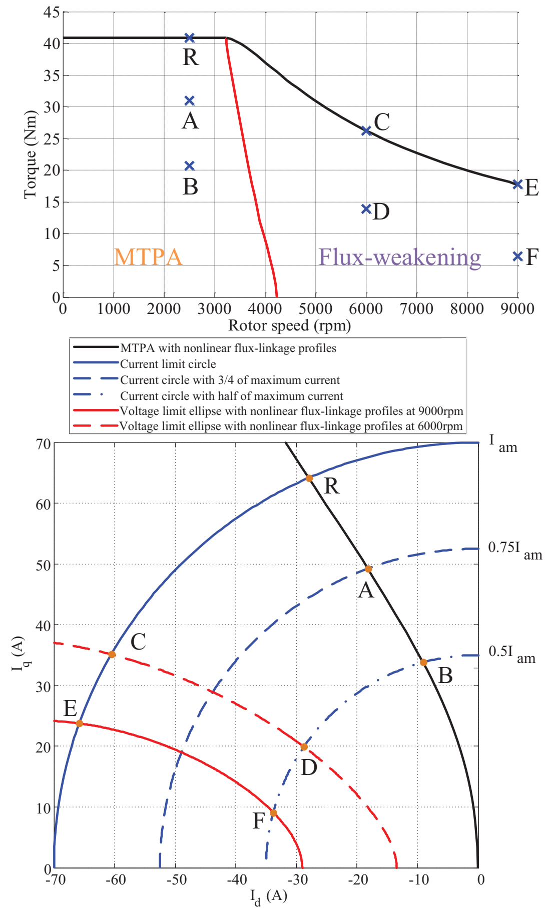

Fig. 4. Selection of operating points for parameters' fitting.

limit circle. Point 'A' is at the intersection of nonlinear MTPA and the current circle with the radius of 3/4 of the current constraint. Points 'B', 'D' and 'F' are on the current circle with half of the maximum phase current. The limited selections are responsible for the flux-linkage mapping, the determination of the slope of the hyperbola curve for MTPA and torque estimation. Solving the optimization problem as shown in (11) leads to the completion of the proposed algorithm.

minimize:

$$
\begin{aligned}
&\sum_{i=1}^{6} w_i\left(\Lambda_{dqi}-\Lambda_{dqim}\right)^2\\
&+w_h\left\{
\begin{aligned}
&-c_3 i_{dh}^3+\left[L_d-L_q+2\left(c_1-c_2\right)i_q\right]i_{dh}^2\\
&+\left[\Lambda_{pm}+2\left(M_{dq}+M_{qd}\right)i_{qh}+2c_3 i_{qh}^2\right]i_{dh}\\
&+\left(c_2-c_1\right)i_{qh}^3+\left(L_q-L_d\right)i_{qh}^2
\end{aligned}
\right\}
\end{aligned}
\tag{11}
$$

subject to

$$
T_{er}-\frac{3}{2}P\left[
\begin{aligned}
&\left(L_d-L_q\right)i_{dr}i_{qr}+\left(c_1-c_2\right)i_{dr}i_{qr}^2\\
&+\left(\Lambda_{pm}-c_3 i_{dr}^2\right)i_{qr}+M_{dq}i_{qr}^2-M_{qd}i_{dr}^2
\end{aligned}
\right]=0,
$$

$$-c_3 i_{dr}^3 + \left[L_d - L_q + 2\left(c_1 - c_2\right) i_{qr}\right] i_{dr}^2 +$$
$$\left[\Lambda_{pm} + 2\left(M_{dq} + M_{qd}\right) i_{qr} + 2c_3 i_{qr}^2\right] i_{dr}$$
$$+\left(c_2 - c_1\right) i_{qr}^3 + \left(L_q - L_d\right) i_{qr}^2 = 0,$$

$$\Lambda_{dr} - (\Lambda_{pm} + L_d i_{dr} + M_{dq} i_{qr} + c_1 i_{dr} i_{qr}) = 0,$$

$$\Lambda_{qr} - (L_q i_{qr} + c_2 i_{qr}^2 + M_{qd} i_{dr} + c_3 i_{dr} i_{qr}) = 0.$$

where,

$$\sum_{i=1}^{6} w_i \left(\Lambda_{dqi} - \Lambda_{dqim}\right)^2 =$$
$$w_{da}\left(\Lambda_{da} - \Lambda_{dam}\right)^2 + w_{qa}\left(\Lambda_{qa} - \Lambda_{qam}\right)^2 + w_{db}\left(\Lambda_{db} - \Lambda_{dbm}\right)^2$$
$$+w_{qb}\left(\Lambda_{qb} - \Lambda_{qbm}\right)^2 + w_{dc}\left(\Lambda_{dc} - \Lambda_{dcm}\right)^2 + w_{qc}\left(\Lambda_{qc} - \Lambda_{qcm}\right)^2$$
$$+w_{dd}\left(\Lambda_{dd} - \Lambda_{ddm}\right)^2 + w_{qd}\left(\Lambda_{qd} - \Lambda_{qdm}\right)^2 + w_{de}\left(\Lambda_{de} - \Lambda_{dem}\right)^2$$
$$+w_{qe}\left(\Lambda_{qe} - \Lambda_{qem}\right)^2 + w_{df}\left(\Lambda_{df} - \Lambda_{dfm}\right)^2 + w_{qf}\left(\Lambda_{qf} - \Lambda_{qfm}\right)^2.$$

w_x is the weight for the corresponding term; Λ_{dx}, Λ_{qx}, Λ_{dxm}, Λ_{qxm} are the real d-axis and q-axis flux-linkages, the d-axis and q-axis flux-linkages calculated by the proposed model at point 'x'; T_{er}, i_{dr}, i_{qr}, Λ_{dr} and Λ_{qr} are the real torque, d-axis and q-axis currents, d-axis and q-axis flux-linkages at point 'R'; T_{eh}, i_{dh}, i_{qh}, Λ_{dh} and Λ_{qh} are the real torque, d-axis and q-axis currents, d-axis and q-axis flux-linkages at point 'B' in Fig. 4.

In general, the constraints of the optimization problem are the torque, MTPA condition, and flux-linkage matching at the rated operating point. The optimization object is to minimize the sum of the weighted MTPA condition at half of the maximum current and the squared errors between the real flux-linkage and the flux-linkage calculated by the proposed model with different weights at the other six points. Eight constant coefficients in (7) can be derived by solving the optimization problem (11) through 'fmincon' in MATLAB. Substituting the resultant constants into (8) and (10) offers the torque estimation and real-time MTPA control, respectively.

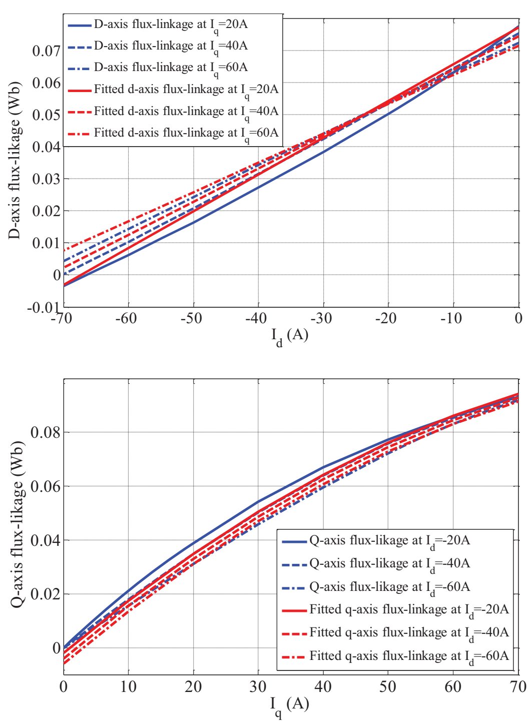

Fig. 5. D-axis and q-axis flux-linkage fittings with the proposed model.

V. Simulation Results

In simulation, the IPM motor's pole pair is 5, the stator resister is 0.078Ω at 100ºC, I_{phase_peak}=70A, and U_{dc}=300V. The flux-linkage profile is obtained from the FEA simulations. The fitting results are listed in Table I.

TABLE I
Coefficient Fitting Results

Coefficient	Value	Unit
Λ_{pm}	0.08	Wb
L_d	0.0013	H
M_{dq}	-1.47×10^{-4}	H
c_1	-6.69×10^{-6}	H/A
L_q	0.0021	H
c_2	-1.01×10^{-5}	H/A
M_{qd}	1.18×10^{-4}	H
c_3	-7.24×10^{-7}	H/A

Fig. 5 shows the d-axis and q-axis flux-linkages obtained from the FEA data in blue and the flux-linkages calculated by the proposed model with the parameters listed in Table I in red. They generally match each other.

Fig. 6 compares the characteristics of an IPM machine between the proposed model and the FEA data. Even though there is some mismatch between the two MTPA curves, Fig. 7 shows that the mismatch does not introduce much more copper loss for the machine.

By using the current references on the MTPA in (5), it describes the comparison between the two types of torque estimations in Section III and Section IV as shown in Fig. 8. The real torque is plotted in black, the torque estimation based on the proposed flux-linkage model is in red, and the torque identification based on the conventional IPMSM model is in blue. For other points, the accuracy improvement of the torque estimation can easily be seen from the comparison between Fig. 2 and Fig. 6.

VI. Conclusion

A new flux-linkage model for IPMSM with constant coefficients is proposed. By using the flux-linkage data, MTPA condition, and torque information at specific seven points, the MTPA and torque fitting can be accomplished. Simulation results indicate the algorithm works well. Compared to the conventional motor model with constant parameters, the proposed flux-linkage model can offer better torque estimation. Besides, the simplicity of the flux-linkage

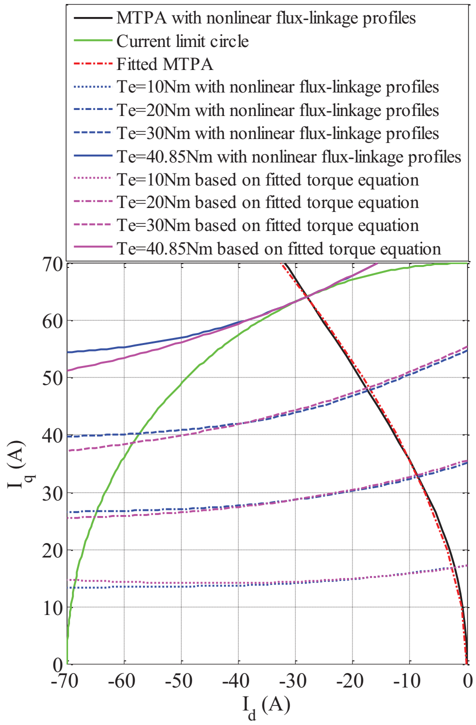

Fig. 6. Characteristic comparison between the proposed model and FEA data.

model enables the implementation of the corresponding MTPA control and torque estimation in real time. The experimental validation will be done for future work.

ACKNOWLEDGMENT

This research was undertaken, in part, thanks to funding from the Canada Excellence Research Chairs Program.

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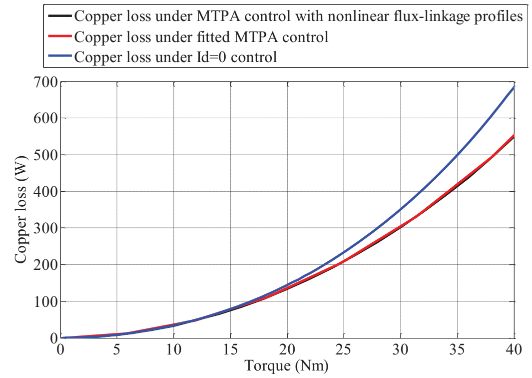

Fig. 7. Copper losses under different control strategies.

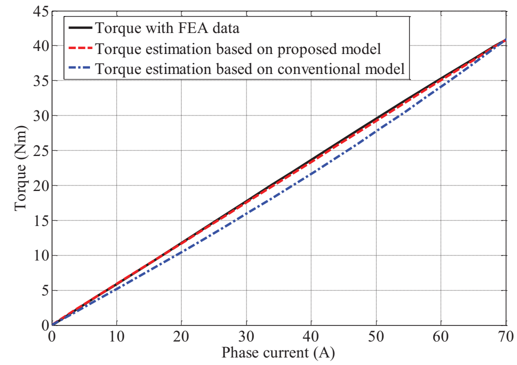

Fig. 8. Torque curves along MTPA with nonlinear flux-linkage profiles.

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